Hamiltonian Analysis of the Deep Inelastic Structure Function

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(31 May 1996)

Abstract

We investigate the feasibility of analyzing deep inelastic structure functions in Hamiltonian formalism by combining the light-front BJL limit of high energy amplitudes and the Fock space (multi-parton) description of hadrons. This study is motivated by some of the theoretical questions emerging from the ongoing nonperturbative/perturbative studies in light-front QCD and also by current problems in the interface of perturbative/nonperturbative QCD. In this preliminary study, we address the unpolarized structure function F_2. Our starting point is the expression for the quark structure function as the Fourier transform of the expectation value of the *good component* of a bilocal current in the target state. By expanding the target state in a set of multi-particle Fock states, the structure function is expressed as the sum of squares of multi-parton wavefunctions integrated over independent longitudinal and transverse parton momenta. Utilizing the fact that the multi-particle Fock states are connected with each other by the light-front QCD Hamiltonian, we study questions of cancellation of collinear singularities, factorization of mass singularities, and the logarithmic scaling violations in the Hamiltonian picture. In this paper the essential features of the formalism are illustrated utilizing the calculation of the structure function of a dressed quark and the evolution of the valence part of the $q\bar{q}$ bound state structure function up to order g^2.

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I. INTRODUCTION

One of the earliest motivations for studying field theories in the light-front formulation came from investigations in current algebra [1]. Also of significant importance were the observation of Bjorken scaling [2] in deep inelastic scattering and the development of Feynman's parton model [3]. At a former level, starting from the algebra of currents in light- front theory and utilizing Bjorken-Johnson-Low (BJL) limit of high energy scattering one could "derive exact scaling" and arrive at an expression for the scaling function as the Fourier transform of a bilocal current matrix element [4]. The structure and consequences of light-front current algebra played a significant role in proposing Quantum Chromodynamics (QCD) as the underlying theory of strong interactions [5]. However, many questions regarding renormalization aspects remained unanswered.

Following the discovery of asymptotic freedom in QCD, scaling and scaling violations were studied [6] utilizing operator product expansions near the light cone and the theory of composite operator renormalization. In this language the emphasis is on manifest gauge and Lorenz covariance and the intuitive parton picture remains hidden. On the other hand, immediately after the discovery of scaling and prior to the discovery of asymptotic freedom, the utility of the infinite momentum frame picture/light-front language in providing simple, intuitive description of high energy phenomena was recognized and exploited by many authors [7–10]. Thus the intuitive approach (physical gauge, light cone variables, ...) to study various issues in deep inelastic scattering have been vigorously pursued [11–15] along with the operator product expansion formalism. In this approach the covariant Feynman Diagram (with four momentum conservation at vertices and off-mass shell particles in intermediate states) is still employed. In contrast is the approach [16,17] using old-fashioned perturbation theory (three momentum conservation at the vertices, on-mass shell particles in intermediate states), light front gauge, and light front variables. As has been stressed many times by Brodsky, Lepage and collaborators [18], it is the latter approach which keeps close contact with the intuitive parton model ideas within field theory.

Old-fashioned perturbation theory in the infinite momentum frame was utilized by Drell, Levy, and Yan [7] to study scaling of the deep inelastic structure function in field theory in the context of pseudoscalar Yukawa model. After the emergence of QCD as the underlying theory of strong interactions, various aspects of structure function have been studied in the old-fashioned light-front perturbation theory formalism. For example, 1) questions concerning the evolution of distributions with emphasis on end-point ($x \rightarrow 1$) behavior [17] (higher twist effects), 2) phenomenological aspects of higher twist effects [19], 3) gluon distribution function [20], 4) photon distribution function [21], and 5) theoretically motivated parameterizations of parton distributions [22].

The main advantages of the Brodsky-Lepage framework may be summarized as follows. The light-front perturbation theory approach is the one closest to the intuitive parton picture, while maintaining the field theoretic aspects of the problem in tact. The framework makes use of the properties of multi-parton wavefunctions which span both the perturbative and non-perturbative sector of the theory. The calculation of certain observables sometimes simplifies drastically compared to the other approaches to the problem. The utility of light front dynamics for the interpretation and the calculation of polarized structure functions and higher twist structure functions in general has been stressed also by Jaffe, Ji and collab-

orators [23]. The disadvantages of the approach are mainly associated with the complexities of renormalization in the Hamiltonian framework. (For early work on renormalization of QED in the infinite momentum frame see, for example, Ref. [24].)

At present, two major approaches aimed at the non-perturbative study of field theories in the light-front formalism [25] are the Discretized Light Cone Quantization (DLCQ) [26] and Light-Front Tamm-Dancoff (LFTD) approximation [27]. Recently there has also been a proposal to study QCD in the light front Hamiltonian approach with particular emphasis on the aspects of renormalization [28]. Two of the most interesting features of light front dynamics in the Hamiltonian language are light front power counting [28,29] and the special aspects of longitudinal dynamics.

Perturbative and non-perturbative aspects of longitudinal dynamics are of interest both from theoretical and phenomenological considerations. The problem associated with exactly zero longitudinal momentum (the so-called "zero-mode" problem) has attracted lot of attention recently. The divergences associated with very small longitudinal momentum also play a major role in the approach of Ref. [28] in the context of incorporating non-perturbative effects in the effective light-front Hamiltonian via perturbation theory framework. Study of deep inelastic structure function in the language of multi-parton wavefunctions offers an opportunity to study directly the question of cancellation/non-cancellation of small longitudinal momentum divergences in physical observables.

At small values of the longitudinal momentum (or equivalently at very small values of x), the long (longitudinal) distance aspects of the theory are probed and one enters the overlapping region of perturbative and non-perturbative dynamics. The small x behavior of structure functions has attracted lot of attention recently in the light of new data from the HERA facility. A recent work of interest is that of Mueller [30] on small x behavior using multi-parton light-front wavefunctions.

In this work we study various aspects of the deep inelastic structure function F_2 utilizing light-front symmetries and multi-parton wavefunctions. As is well known, some aspects of the problem are easily probed in coordinate space and some aspects are easily probed in momentum space. A convenient starting point to discuss features of deep inelastic scattering in the Hamiltonian framework is provided by the expression for the structure function as the Fourier transform of a bilocal matrix element. Among the many derivations of this result that exist in the literature, is the one utilizing equal-x^+ light-front current algebra and the BJL limit [4]. In this approach (which closely follows the original derivation of Bjorken in the infinite momentum frame) partons are not introduced from the beginning and "perfect" scaling emerges (after canonical manipulations) as a result of special properties of the equal x^+ commutators of currents. One essential feature of the commutators is the non-local behavior in the longitudinal direction which is a consequence of the symmetries on the light-front. Furthermore, the current commutators are found to have the same *form* in free and interacting theories. Thus BJL limit together with light-front current algebra allows us to have a starting point (which is very close to the physical answer) for the Hamiltonian analysis of the structure function..

To address the problem of scaling violations in the Hamiltonian picture and the associated renormalization, it is convenient to introduce the constituents of the hadron. As is well known, one can make the synthesis between the current algebra picture and the parton picture by expanding the state in terms of multi-parton (constituent) wavefunctions [31].

The partons that appear in this formalism need not be collinear or massless but they are always on mass shell since we are using the Hamiltonian dynamics. The F_2 structure function appears as the sum of squares of multi-parton wavefunctions integrated over independent longitudinal and transverse momenta. The behavior of multi-parton wavefunctions is determined by the light-front Hamiltonian. In this picture scaling violations are intimately associated with the renormalization aspects of multi-parton wavefunctions.

Apart from the issues of renormalization, there is a question that confronts any approach that attempts the description of structure function in terms of constituent wavefunctions. Is it possible, even in principle, to describe the hadron structure function in terms of a finite number of constituent wave functions, since " the average number is infinite" [3,32] according to the standard folklore. What we have to worry about are the gluon and sea distribution functions and their physical interpretation. To address this question, we need to take into consideration, their definition in terms of the multi-parton wavefunctions, various physical constraints from the normalization of the hadron state, longitudinal momentum sum rule, etc. The approach using parton wavefunctions offers the possibility to address this question.

It is of interest also to see how various issues like the cancellation of collinear singularities, factorization (separation of soft and hard physical processes), suppression of coherent (interference) effects in the hard processes, scale evolution of parton densities, etc in perturbative QCD emerge in the present formalism.

The plan of this paper is as follows. In Sec. II we discuss the general feature of the structure function F_2. Structure function of a dressed quark is discussed in Sec. III to illustrate the calculational framework. In Section IV we discuss the structure function of a meson-like bound state. Logarithmic scaling violations in the bound state structure function are discussed in Section V. Discussion and summary is presented in Sec. VI. Finally the details of the normalization of the state and longitudinal momentum fraction sum rules are discussed in appendices A and B respectively.

II. UNPOLARIZED STRUCTURE FUNCTION F_2

We start from the expression for the F_2 structure function (more precisely, the quark momentum fraction distribution function):

$$\frac{F_2(x)}{x} = \frac{i}{2\pi} \int d\eta e^{-i\eta x} \bar{V}^1(\eta). \tag{2.1}$$

The generalized form factor $\bar{V}^1$ is defined by

$$\langle P \mid \bar{\mathcal{J}}^+(y \mid 0) \mid P \rangle = P^+ \bar{V}^1(\eta), \tag{2.2}$$

where

$$\bar{\mathcal{J}}^+(y \mid 0) = \frac{1}{2i} [\bar{\psi}(y)\gamma^+\psi(0) - \bar{\psi}(0)\gamma^+\psi(y)]. \tag{2.3}$$

The bilocality is only in the longitudinal direction, *i.e.,* $y^+ = 0, y^\perp = 0$. Note that $x = -\frac{q^+}{P^+}$ and $\eta = \frac{1}{2}P^+ y^-$. The field operator $\psi^+(y)$ is given by [33]

$$\psi^+(y) = \begin{pmatrix} \xi(y) \\ 0 \end{pmatrix} \tag{2.4}$$

where

$$\xi(y) = \sum_{s_1} \chi_{s_1} \int [dp_1][b(p_1, s_1) e^{-\frac{i}{2} p_1^+ y^-} + d^\dagger(p_1, -s_1) e^{\frac{i}{2} p_1^+ y^-}], \tag{2.5}$$

with $[dp_1]$ defined in appendix A. In this framework, the operator has a simple structure and all the complexities are buried in the state $\mid P\rangle$. The state $\mid P\rangle$ can be expanded in terms of multi-parton Fock space amplitudes which are related to each other through the relativistic light-front version of the Schroedinger equation

$$P^- \mid P\rangle = \frac{M^2 + (P^\perp)^2}{P^+} \mid P\rangle, \tag{2.6}$$

where P^- is the light-front QCD Hamiltonian which can be written as

$$P^- = P_0^- + V \tag{2.7}$$

with P_0^- is the free part and V is the interaction part. The explicit expressions for P_o^- and V are given in Ref. [33]. We note that the main focus of the ongoing effort in light-front theory is to solve this equation nonperturbatively. We can explore various features of the structure function utilizing the light-front Hamiltonian P^-.

When we address the problem of calculating the structure function of a composite system like a baryon or a meson, which of course are the physically interesting systems, we are immediately bombarded with all the complexities in the real world. To break up the problem in to simpler pieces, in the following section we first consider an artificial target, namely, a dressed parton. Even though such an object does not appear in the real world, this calculation will help us to understand some of the key issues in a simpler setting and also to set the notation. For calculations of structure function of a quark done in the framework of Feynman perturbation theory in planar gauge see Dokshitzer, Dyakonov, and Troyan [13].

III. STRUCTURE FUNCTION OF A DRESSED QUARK

A. Parton picture

We take the state $\mid P\rangle$ to be a dressed quark and expand this state in terms of bare states of quark, quark plus gluon, quark plus two gluons, etc. The expansion takes the form

$$\begin{aligned}
\mid P\sigma\rangle = {} & \phi_1(P,\sigma) b^\dagger(P,\sigma) \mid 0\rangle \\
& + \sum_{\sigma_1,\lambda_2} \int \{dk_1\} \int \{dk_2\} \\
& \phi_2(P,\sigma \mid k_1,\sigma_1; k_2,\lambda_2) \sqrt{2(2\pi)^3 P^+} \delta^3(P - k_1 - k_2) b^\dagger(k_1,\sigma_1) a^\dagger(k_2,\lambda_2) \mid 0\rangle \\
& + \frac{1}{2} \sum_{\sigma_1,\lambda_2,\lambda_3} \int \{dk_1\} \int \{dk_2\} \int \{[dk_3\} \\
& \phi_3(P,\sigma \mid k_1,\sigma_1; k_2,\lambda_2; k_3,\lambda_3) \sqrt{2(2\pi)^3 P^+} \delta^3(P - k_1 - k_2 - k_3) \\
& \quad b^\dagger(k_1,\sigma 1) a^\dagger(k_2,\lambda_2) a^\dagger(k_3,\lambda_3) \mid 0\rangle \\
& + \quad \ldots.
\end{aligned} \tag{3.1}$$

The factor of $\frac{1}{2}$ in front of the expression in the third term above is the symmetry factor for identical bosons.

The function $\phi(P, \sigma)$ is the probability amplitude to find a bare quark with momentum P and helicity σ in a dressed quark, the function $\phi(P, \sigma \mid k_1\sigma_1, k_2\lambda_2)$ is the probability amplitude to find a bare quark with momentum k_1 and helicity σ_1 and a bare gluon with momentum k_2 and helicity λ_2 in the dressed quark, etc.

We evaluate the expression in Eq.(2.1) explicitly, noting that in the present case the contribution from the second term in Eq.(2.3) is zero. Introduce the Jacobi momenta $(x_i, \kappa_i^\perp)$

$$k_i^+ = x_i P^+,\ k_i^\perp = \kappa_i^\perp + x_i P^\perp \tag{3.2}$$

so that

$$\sum_i x_i = 1, \sum_i \kappa_i^\perp = 0. \tag{3.3}$$

Also introduce the amplitudes,

$$\begin{aligned} \phi_1 &= \psi_1, \\ \phi_2(k_i^+, k_i^\perp) &= \frac{1}{\sqrt{P^+}} \psi_2(x_i, \kappa_i^\perp), \\ \phi_3(k_i^+, k_i^\perp) &= \frac{1}{P^+} \psi_3(x_i, \kappa_i^\perp), \end{aligned} \tag{3.4}$$

and so on. We get

$$\begin{aligned} \frac{F_2(x)}{x} &= \mid \psi_1 \mid^2 \delta(1-x) \\ &+ \sum_{\sigma_1,\lambda_2} \int dx_2 \int d^2\kappa_1^\perp \int d^2\kappa_2^\perp \delta(1-x-x_2)\delta^2(\kappa_1^\perp + \kappa_2^\perp) \mid \psi_2^{\sigma_1,\lambda_2}(x, \kappa_1^\perp; x_2, \kappa_2^\perp) \mid^2 \\ &+ \frac{1}{2} \sum_{\sigma_1,\lambda_2,\lambda_3} \int dx_2 \int dx_3 \int d^2\kappa_1^\perp \int d^2\kappa_2^\perp \int d^2\kappa_3^\perp \delta(1-x-x_2-x_3)\delta^2(\kappa_1^\perp + \kappa_2^\perp + \kappa_3^\perp) \\ &\qquad \mid \psi_3^{\sigma_1\lambda_2\lambda_3}(x, \kappa_1^\perp; x_2, \kappa_2^\perp; x_3, \kappa_3^\perp) \mid^2 \ + ... \end{aligned} \tag{3.5}$$

This equation makes manifest the partonic interpretation of the quark distribution function, namely, the quark distribution function of a dressed quark is the incoherent sum of probabilities to find a bare parton (quark) with longitudinal momentum fraction x in various multi-particle Fock states of the dressed quark. Since we have computed the distribution function in field theory, there are also significant differences from the traditional parton model [3]. Most important difference is the fact that the partons in field theory have transverse momenta ranging from zero to infinity. Whether the structure function scales or not now depends on the ultraviolet behavior of the multi-parton wavefunctions. By analyzing various interactions, one easily finds that in superrenormalizable interactions, the transverse momentum integrals converge in the ultraviolet and the structure function scales, whereas in renormalizable interactions, the transverse momentum integrals diverge in the ultraviolet which in turn leads to scaling violations in the structure function.

We should also note that even though (3.5) looks like an incoherent sum, interference effects are also present in this expression. We will elaborate more on this in Sec.V.

B. Dressing with one gluon

In this section we evaluate the structure function of a dressed quark to order α_s. Our starting point is the eigenvalue equation obeyed by the dressed quark state, *i.e.,* Eq.(2.6). We substitute the Fock space expansion (3.1) in (2.6) and make projection with a bare one quark - one gluon state $b^\dagger(p_1, s_1)a^\dagger(p_2, \rho_2) \mid 0\rangle$ keeping only terms up to order g. We arrive at

$$[\frac{M^2 + (P^\perp)^2}{P^+} - \frac{m^2 + (p_1^\perp)^2}{p_1^+} - \frac{(p_2^\perp)^2}{p_2^+}]\phi_2(P, \sigma \mid p_1, s_1; p_2, \rho_2) =$$
$$- \frac{g}{\sqrt{2(2\pi)^3}} T^a \frac{1}{\sqrt{p_2^+}} \chi_{s_1}^\dagger [2\frac{p_2^\perp}{p_2^+} - \frac{\sigma^\perp . p_1^\perp - im}{p_1^+}\sigma^\perp - \sigma^\perp \frac{\sigma^\perp . P^\perp + im}{P^+}] \chi_\sigma . (\epsilon_{\rho_2}^\perp)^* \; \phi_1(P, \sigma). \quad (3.6)$$

We rewrite the above equation in terms of Jacobi momenta ($p_i^+ = x_i P^+, \kappa_i^\perp = p_i^\perp + x_i P^\perp$) and the wavefunctions ψ_i which are functions of Jacobi momenta. Using the notation $x = x_1, \kappa_1 = \kappa$ and using the facts $x_1 + x_2 = 1, \kappa_1 + \kappa_2 = 0$, we have

$$[M^2 - \frac{m^2 + (\kappa^\perp)^2}{x} - \frac{(\kappa^\perp)^2}{1 - x}]\psi_2^{s_1, \rho21}(x, \kappa^\perp; 1 - x, -\kappa^\perp) =$$
$$- \frac{g}{\sqrt{2(2\pi)^3}} T^a \frac{1}{\sqrt{1 - x}} \chi_{s_1}^\dagger [- 2\frac{\kappa^\perp}{1 - x} - \frac{\sigma^\perp . \kappa^\perp - im}{x}\sigma^\perp - \sigma^\perp im] \chi_\sigma . (\epsilon_{\rho_2}^\perp)^* \psi_1 \quad (3.7)$$

Taking the bare and dressed quarks to be massless, we arrive at

$$\sum_{\sigma_1, \rho_2} \int d^2\kappa^\perp \mid \psi_2^{\sigma_1, \rho_2}(x, \kappa^\perp, 1 - x, -\kappa^\perp) \mid^2 = \frac{g^2}{(2\pi)^3} C_f \mid \psi_1 \mid^2 \frac{1 + x^2}{1 - x} \int d^2\kappa^\perp \frac{1}{(\kappa^\perp)^2} \quad (3.8)$$

where $C_f = \frac{N^2 - 1}{2N}$. The transverse momentum integral is divergent at both limits of integration. We regulate the lower limit by μ and the upper limit by Q. Thus we have

$$\frac{F_2(x)}{x} = \mid \psi_1 \mid^2 [\delta(1 - x) \; + \; \frac{\alpha_s}{2\pi} C_f \frac{1 + x^2}{1 - x} ln \frac{Q^2}{\mu^2}]. \quad (3.9)$$

The normalization condition reads

$$\mid \psi_1 \mid^2 [1 + \frac{\alpha_s}{2\pi} C_f \int dx \frac{1 + x^2}{1 - x} ln \frac{Q^2}{\mu^2}] = 1. \quad (3.10)$$

Within the present approximation (valid only upto α_s),

$$\mid \psi_1 \mid^2 = 1 - \frac{\alpha_s}{2\pi} C_f \int dx \frac{1 + x^2}{1 - x} ln \frac{Q^2}{\mu^2}. \quad (3.11)$$

In the second term we recognize the familiar expression of wavefunction correction of the state n in old fashioned perturbation theory, namely, $\sum_m' \frac{|\langle m|V|n\rangle|^2}{(E_n - E_m)^2}$.

Thus to order α_s,

$$\frac{F_2(x)}{x} = \delta(1 - x) + \frac{\alpha_s}{2\pi} C_f ln \frac{Q^2}{\mu^2} [\frac{1 + x^2}{1 - x} - \delta(1 - x) \int dy \frac{1 + y^2}{1 - y}]. \quad (3.12)$$

Note that (3.12) can also be written as

$$\frac{F_2(x)}{x} = \delta(1-x) + \frac{\alpha_s}{2\pi} C_f ln \frac{Q^2}{\mu^2} [\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)], \tag{3.13}$$

which is a more familiar expression. Note that by construction $\mid \psi_2(x, \kappa^\perp) \mid^2$ is a probability density. However, this function is singular as $x \to 1$ (gluon longitudinal momentum fraction approaching zero). To get a finite probability density we have to introduce a cutoff ϵ ($x_{gluon} > \epsilon$), for example. In a physical cross section, this ϵ cannot appear and here we have an explicit example of this cancellation. The probabilistic interpretation of the splitting function $P_{qq} = \frac{1+x^2}{1-x}$ which arise from real gluon emission is obvious in our derivation. On the other hand, the function $\tilde{P}_{qq} = \frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)$ doesn't have the probabilistic interpretation since it includes contribution from virtual gluon emission. This is immediately transparent from the relation

$$\int dx \tilde{P}_{qq}(x) = 0. \tag{3.14}$$

We also note that the divergence arising from small transverse momentum (the familiar mass singularity) cannot be handled properly in the present calculation. This is to be contrasted with the calculation of the meson structure function (see the following section) where the mass singularities can be properly absorbed into the nonperturbative part of the structure function.

It is instructive to compare the above derivation with typical calculations that exist in the literature. Conventionally, one evaluates the imaginary part of the forward virtual Compton amplitude order by order in perturbation theory. In the lowest order, in addition to the real gluon emission, one also has various self-energy diagrams and the dressing of the photon vertex by the gluon. The ultraviolet part of the vertex and parts of the self-energy contributions cancel each other as a result of the QED Ward Identity $Z_1 = Z_2$. Thus, for example, in planar gauge, Dokshitzer, Dyakonov and Troyan [13] find by explicit calculations that neither collinear nor infrared mass singularities affect the "partonometric vertex" and that the radiative corrections to the photon vertex appear to be effectively unity. By explicit calculations both in the ϕ^3 theory and QCD, Collins, Soper, and Sterman [34] find that non-ladder diagrams are either higher twist or contribute only to the hard part. In the calculation of Brodsky and Lepage [17] using light-front time ordered perturbation theory in the gauge $A^+ = 0$, the identity $Z_1 = Z_2$ is used. In the present calculation, since we have started from the BJL limit, dressing of the photon vertex, gluon radiation from the final state, and parts of self-energy contributions (that cancel eventually) does not appear at all which leads to simplification of the calculation. It is of interest to see whether such simplifications persist to higher orders in the present framework.

IV. STRUCTURE FUNCTION OF A MESON-LIKE BOUND STATE

For simplicity we consider a meson-like bound state. We expand the state $\mid P\rangle$ for $q\bar{q}$ bound state in terms of the Fock components $q\bar{q}$, $q\bar{q}g$, ... as follows.

$$
\begin{aligned}
\mid P\rangle = &\sum_{\sigma_1,\sigma_2} \int [dk_1] \int [dk_2] \\
& \phi_2(P \mid k_1,\sigma_1;k_2,\sigma_2)\sqrt{2((2\pi)^3 P^+}\delta^3(P-k_1-k_2) b^\dagger(k_1,\sigma_1) d^\dagger(k_2,\sigma_2) \mid 0\rangle \\
& + \sum_{\sigma_1,\sigma_2,\lambda_3} \int \{dk_1\} \int \{dk_2\} \int \{dk_3\} \\
& \phi_3(P \mid k_1,\sigma_1;k_2,\sigma_2;k_3,\lambda_3)\sqrt{2(2\pi)^3 P^+}\delta^3(P-k_1-k_2-k_3) \\
& \quad b^\dagger(k_1,\sigma_1) d^\dagger(k_2,\sigma_2) a^\dagger(k_3,\lambda_3) \mid 0\rangle \\
& + \ ... \ .
\end{aligned} \tag{4.1}
$$

Here ϕ_2 is the probability amplitude to find a quark and an antiquark in the meson, ϕ_3 is the probability amplitude to find a quark, antiquark and a gluon in the meson etc.

As in Sec. III we evaluate the expression in Eq.(2.1) explicitly. The contribution from the first term (from the quark), in terms of the amplitudes

$$
\begin{aligned}
\phi_2(k_i^+, k_i^\perp) &= \frac{1}{\sqrt{P^+}}\psi_2(x_i,\kappa_i^\perp), \\
\phi_3(k_i^+, k_i^\perp) &= \frac{1}{P^+}\psi_3(x_i,\kappa_i^\perp),
\end{aligned} \tag{4.2}
$$

and so on, is

$$
\begin{aligned}
\frac{F_2(x)^q}{x} = &\sum_{\sigma_1,\sigma_2} \int dx_2 \int d^2\kappa_1^\perp \int d^2\kappa_2^\perp \delta(1-x-x_2)\delta^2(\kappa_1+\kappa_2) \mid \psi_2^{\sigma_1,\sigma_2}(x,\kappa_1^\perp;x_2\kappa_2^\perp) \mid^2 \\
& + \sum_{\sigma_1,\sigma_2,\lambda_3} \int dx_2 \int dx_3 \int d^2\kappa_1^\perp \int d^2\kappa_2^\perp \int d^2\kappa_3^\perp \delta(1-x-x_2-x_3)\delta^2(\kappa_1+\kappa_2+\kappa_3) \\
& \qquad \mid \psi_3^{\sigma_1,\sigma_2,\lambda_3}(x,\kappa_1^\perp;x_2,\kappa_2^\perp;x_3,\kappa_3^\perp) \mid^2 + ... \ .
\end{aligned} \tag{4.3}
$$

Again, the partonic interpretation of the F_2 structure function is manifest in this expression.

Contributions to the structure function from the second term in Eq.(2.1) is

$$
\begin{aligned}
\frac{F_2(x)^{\bar q}}{x} = &\sum_{\sigma_1,\sigma_2} \int dx_2 \int d^2\kappa_1^\perp \int d^2\kappa_2^\perp \delta(1-x-x_2)\delta^2(\kappa_1+\kappa_2) \mid \psi_2^{\sigma_1,\sigma_2}(x_2,\kappa_2^\perp;x,\kappa_1^\perp) \mid^2 \\
& + \sum_{\sigma_1,\sigma_2,\lambda_3} \int dx_2 \int dx_3 \int d^2\kappa_1^\perp \int d^2\kappa_2^\perp \int d^2\kappa_3^\perp \delta(1-x-x_2-x_3)\delta^2(\kappa_1+\kappa_2+\kappa_3) \\
& \qquad \mid \psi_3^{\sigma_1,\sigma_2,\lambda_3}(x_2,\kappa_2^\perp;x,\kappa_1^\perp;x_3,\kappa_3^\perp) \mid^2 + ... \ .
\end{aligned} \tag{4.4}
$$

The normalization condition given in Eq.(B3) guarantees that

$$
\int dx [\frac{F_2^q(x)}{x} + \frac{F_2^{\bar q}(x)}{x}] = 2 \tag{4.5}
$$

which reflects the fact that there are two valence particles in the meson. Since the bilocal current component $\bar{\mathcal{J}}^+$ involves only fermions explicitly, we appear to have missed the contributions from the gluon constituents altogether. Gluonic contribution to the structure function F_2 is most easily calculated by studying the hadron expectation value of the conserved longitudinal momentum operator P^+. The details are given in appendix C.

From the normalization condition, it is clear that the valence distribution receives contribution from the amplitudes ψ_2, ψ_3, ... at any scale μ. This has interesting phenomenological implications. In the model for the meson with only a quark-antiquark pair of equal mass, the valence distribution function will peak at $x = \frac{1}{2}$. If there are more than just the two particles in the system, the resulting valence distribution will naturally have an enhancement in the $x < \frac{1}{2}$ region and a depletion in the $x > \frac{1}{2}$ region, as a simple consequence of longitudinal momentum conservation.

The equation (4.3) as it stands is useful only when the bound state solution in QCD is known in terms of the multi-parton wavefunctions. The wavefunctions, as they stand, span both the perturbative and non-perturbative sectors of the theory. Great progress in the understanding of QCD in the high energy sector is made in the past by separating the soft (non-perturbative) and hard (perturbative) regions of QCD via the machinery of factorization. It is of interest to see under what circumstances a factorization occurs in the formal result of Eq. (4.3) and a perturbative picture of scaling violations emerges finally. We address this issue in the following section.

V. PERTURBATIVE PICTURE OF SCALING VIOLATIONS

To address the issue of scaling violations in the structure function of the "meson-like" bound state, it is convenient to separate the momentum space into low-energy and high-energy sectors. Such a separation has been introduced in the past in the study of renormalization aspects of bound state equations [35] in light-front field theory. The two sectors are formally defined by introducing cutoff factors in the momentum space integrals. How to cutoff the momentum integrals in a sensible and convenient way in light-front theory is a subject under active research at the present time. Complications arise because of the possibility of large energy divergences from both small k^+ and large $k^\perp$ regions. In the following we investigate only the effects of logarithmic divergences arising from large transverse momenta, ignore subtelities arising from both small x ($x \rightarrow 0$) and large x ($x \rightarrow 1$) regions and subsequently use simple transverse momentum cutoff. For complications arising from $x \rightarrow 1$ region see Ref. [17].

A. Scale separation

We define the soft region to be $\kappa^\perp < \mu$ and the hard region to be $\mu < \kappa^\perp < \Lambda$. μ serves as a factorization scale which separates soft and hard regions. Since it is an intermediate scale introduced artificially purely for convenience, physical structure function should be independent of μ. The multi-parton amplitude ψ_2 is a function of a single relative transverse momentum $\kappa_1^\perp$ and we define

$$\psi_2 = \begin{cases} \psi_2^s, & 0 < \kappa_1^\perp < \mu, \\ \psi_2^h, & \mu < \kappa_1^\perp < \Lambda \end{cases} \tag{5.1}$$

The amplitude ψ_3 is a function of two relative momenta, $\kappa_1^\perp$ and $\kappa_2^\perp$ and we define

$$\psi_3 = \begin{cases} \psi_3^{ss},\ 0 < \kappa_1^\perp, \kappa_2^\perp < \mu \\ \psi_3^{sh},\ 0 < \kappa_1^\perp < \mu, \\ \psi_3^{hs},\ \mu < \kappa_1^\perp < \Lambda,\ 0 < \kappa_2^\perp < \mu, \\ \psi_3^{hh},\ \mu < \kappa_1^\perp, \kappa_2^\perp < \Lambda \end{cases} \tag{5.2}$$

Let us consider the quark distribution function $q(x) = \frac{F_2(x)}{x}$ defined in Eq.(4.3). In presence of the ultraviolet cutoff Λ, $q(x)$ depends on Λ and schematically we have,

$$q(x, \Lambda^2) = \sum \int_0^\Lambda \psi_2^2 + \sum \int_0^\Lambda \int_0^\Lambda \psi_3^2. \tag{5.3}$$

For convenience, we write,

$$q(x, \Lambda^2) = q_2(x, \Lambda^2) + q_3(x, \Lambda^2). \tag{5.4}$$

where the subscripts 2 and 3 denotes the two-particle and three-particle contributions respectively. Schematically we have,

$$\begin{aligned} q(x, \Lambda^2) = q(x, \mu^2) &+ \sum \int_\mu^\Lambda \mid \psi_2^h \mid^2 \\ &+ \sum \int_0^\mu \int_\mu^\Lambda \mid \psi_3^{sh} \mid^2 + \sum \int_\mu^\Lambda \int_0^\mu \mid \psi_3^{hs} \mid^2 \\ &+ \sum \int_\mu^\Lambda \int_\mu^\Lambda \mid \psi_3^{hh} \mid^2 . \end{aligned} \tag{5.5}$$

We investigate the contributions from the amplitudes ψ_3^{sh} and ψ_3^{hs} to order α_s in the following.

B. Dressing with one gluon

We substitute the Fock expansion Eq. (4.1) in Eq. (2.6) and make projection with a three particle state $b^\dagger(k_1, \sigma_1) d^\dagger(k_2, \sigma_2) a^\dagger(k_3, \sigma_3) \mid 0\rangle$ from the left. In terms of the amplitudes ψ_2, ψ_3, we get,

$$\psi_3^{\sigma_1 \sigma_2 \lambda_3}(x, \kappa_1; x_2, \kappa_2; 1 - x - x_2, \kappa_3) = \mathcal{M}_1 + \mathcal{M}_2, \tag{5.6}$$

where

$$\mathcal{M}_1 = \frac{1}{E}(-)\frac{g}{\sqrt{2(2\pi)^3}} T^a \frac{1}{\sqrt{1 - x - x_2}}\ V_1\ \psi_2^{\sigma_1' \sigma_2}(1 - x_2, -\kappa_2^\perp; x_2, \kappa_2^\perp), \tag{5.7}$$

and

$$\mathcal{M}_2 = \frac{1}{E}\frac{g}{\sqrt{2(2\pi)^3}} T^a \frac{1}{\sqrt{1 - x - x_2}}\ V_2\ \psi_2^{\sigma_1 \sigma_2'}(x, \kappa_1^\perp; 1 - x, -\kappa_1^\perp) \tag{5.8}$$

with

$$E = [M^2 - \frac{m^2 + (\kappa_1^\perp)^2}{x} - \frac{m^2 + (\kappa_2^\perp)^2}{x_2} - \frac{(\kappa_3^\perp)^2}{1-x-x_2}], \tag{5.9}$$

$$V_1 = \chi^\dagger_{\sigma_1} \sum_{\sigma_1'} [\frac{2\kappa_3^\perp}{1-x-x_2} - \frac{(\sigma^\perp.\kappa_1^\perp - im)}{x}\sigma^\perp + \sigma^\perp \frac{(\sigma^\perp.\kappa_2^\perp - im)}{1-x_2}]\chi_{\sigma_1'}.(\epsilon^\perp_{\lambda_1})^*, \tag{5.10}$$

and

$$V_2 = \chi^\dagger_{-\sigma_2} \sum_{\sigma_2'} [\frac{2\kappa_3^\perp}{1-x-x_2} - \sigma^\perp \frac{(\sigma^\perp.\kappa_2^\perp - im)}{y} + \sigma^\perp \frac{(\sigma^\perp.\kappa_1^\perp - im)}{1-x}\sigma^\perp]\chi_{-\sigma_2'}.(\epsilon^\perp_{\lambda_1})^* \tag{5.11}$$

C. Perturbative analysis

For $\kappa_1^\perp$ hard and $\kappa_2^\perp$ soft, $\kappa_1^\perp + \kappa_2^\perp \approx \kappa_1^\perp$ and the multiple transverse momentum integral over ψ_3 factorises into two independent integrals and the longitudinal momentum fraction integrals become convolutions. The contribution from $\mathcal{M}_1$ to ψ_3 is,

$$\begin{aligned}\psi_{3,1}^{\sigma_1,\sigma_2,\Lambda_3}(x,\kappa_1^\perp; x_2,\kappa_2^\perp; 1-x-x_2, -\kappa_2^\perp) = -\frac{g}{\sqrt{2(2\pi)^3}}T^a \frac{x\sqrt{1-x-x_2}}{1-x_2}\frac{1}{(\kappa_1^\perp)^2} \\ \chi^\dagger_{\sigma_1}\sum_{\sigma_1'}[\frac{2\kappa_1^\perp}{1-x-x_2} - \frac{\sigma^\perp.\kappa_1^\perp}{x}\sigma^\perp]\chi'_{\sigma_1}.(\epsilon^\perp_{\lambda_1})^* \\ \psi_2^{\sigma_1',\sigma_2}(1-x_2, -\kappa_2^\perp; x_2, \kappa_2^\perp).\end{aligned} \tag{5.12}$$

Thus the contribution from $\mathcal{M}_1$ to the structure function is

$$\sum \int \mid \psi_{3,1}^{hs} \mid^2 = \frac{\alpha_s}{2\pi} C_f ln\frac{\Lambda^2}{\mu^2}\int_x^1 \frac{dy}{y} P_{qq}(\frac{x}{y}) q_2(y,\mu^2), \tag{5.13}$$

where

$$P_{qq}(\frac{x}{y}) = \frac{1+(\frac{x}{y})^2}{1-\frac{x}{y}}. \tag{5.14}$$

For the configuration $\kappa_1^\perp$ hard, $\kappa_2^\perp$ soft, contribution from $\mathcal{M}_2$ does not factorize and the asymptotic behavior of the integrand critically depends on the asymptotic behavior of the two-particle wavefunction ψ_2. To determine this behavior, we have to analyze the bound state equation which shows that for large transverse momentum $\psi_2(\kappa^\perp) \approx \frac{1}{(\kappa^\perp)^2}$. Thus contribution from $\mathcal{M}_2$ for scale evolution is suppressed by the bound state wavefunction. Analysis of the interference terms (between $\mathcal{M}_1$ and $\mathcal{M}_2$) shows that their contribution also is suppressed by the bound state wavefunction.

For the configuration $\kappa_1^\perp$ soft, $\kappa_2^\perp$ hard, contributions from $\mathcal{M}_1$ and the interference terms are suppressed by the wave function. Contribution from $\mathcal{M}_2$ factorises both in transverse and longitudinal space and generate a pure wavefunction renormalization contribution:

$$\sum \int \mid \psi_{3,2}^{sh} \mid^2 = \frac{\alpha_s}{2\pi} C_f ln\frac{\Lambda^2}{\mu^2}\int_0^1 dy \frac{1+y^2}{1-y} q_2(x,\mu^2). \tag{5.15}$$

We have seen that even though the multi-parton contributions to the structure function involve both coherent and incoherent phenomena, in the hard region coherent effects are suppressed by the wavefunction.

D. Corrections from normalization condition

In the dressed quark calculation, we have seen that the singularity that arises as $x \to 1$ from real gluon emission is canceled by the correction from the normalization of the state (virtual gluon emission contribution from wave function renormalization). In the meson bound state calculation, so far we have studied the effects of a hard real gluon emission. In this section we study the corrections arising from the normalization condition of the quark distribution in the composite bound state.

Collecting all the terms arising from the hard gluon emission contributing to the quark distribution function, we have,

$$\begin{aligned} q(x,\Lambda^2) = & \; q_2(x,\mu^2) + q_3(x,\mu^2) \\ & + \frac{\alpha_s}{2\pi} C_f ln \frac{\Lambda^2}{\mu^2} \int_x^1 \frac{dy}{y} P_{qq}(\frac{x}{y}) q_2(y,\mu^2) \\ & + \frac{\alpha_s}{2\pi} C_f ln \frac{\Lambda^2}{\mu^2} q_2(x,\mu^2) \int dy P(y). \end{aligned} \tag{5.16}$$

We have a similar expression for the antiquark distribution function.

The normalization condition on the quark distribution function should be such that there is one valence quark in the bound state at any scale Q. We choose the factorization scale $\mu = Q_0$. Let us first set the scale $\Lambda = Q_0$. Then we have (in the truncated Fock space)

$$\int_0^1 dx q_2(x,Q_0^2) + \int_0^1 dx q_3(x,Q_0^2) = 1. \tag{5.17}$$

Next set the scale $\Lambda = Q$. We still require

$$\int_0^1 dx q_2(x,Q^2) + \int_0^1 dx q_3(x,Q^2) = 1. \tag{5.18}$$

We note that the evolution of q_3 requires an extra hard gluon which is not available in the truncated Fock space. Thus in the present approximation $q_3(x,Q^2) = q_3(x,Q_0^2)$.

Carrying out the integration explicitly, we arrive at

$$\int_0^1 dx q_2(x,\mu^2)[1 + \frac{2\alpha_s}{2\pi} C_f ln \frac{Q^2}{Q_0^2} \int dy P(y)] + \int_0^1 dx q_3(x,Q^2) = 1 \tag{5.19}$$

Thus we face the necessity to "renormalize" our quark distribution function. Let us define a renormalized quark distribution function

$$q_2^R(x,Q_0^2) = q_2(x,Q_0^2)[1 + \frac{\alpha_s}{2\pi} C_f ln \frac{Q^2}{Q_0^2} \int_0^1 dy P(y)] \tag{5.20}$$

so that, to order α_s,

$$\int_0^1 dx q_2^R(x,Q_0^2) + \int_0^1 dx q_3(x,Q_0^2) = 1. \tag{5.21}$$

We have,

$$q_2(x,Q_0^2) = q_2^R(x,Q_0^2)[1 - 2\frac{\alpha_s}{2\pi}C_f ln\frac{Q^2}{Q_0^2}\int_0^1 dyP(y)]. \tag{5.22}$$

Collecting all the terms, to order α_s, we have the normalized quark distribution function,

$$\begin{aligned} q(x,Q^2) =& \, q_2^R(x,Q_0^2) \\ &+ \frac{\alpha_s}{2\pi}C_f ln\frac{Q^2}{Q_0^2}\int_0^1 dy q_2^R(y,Q_0^2)\int_0^1 dz\delta(zy-x)\tilde{P}(z) \\ &+ q_3(x,Q^2) \end{aligned} \tag{5.23}$$

with $\tilde{P}(z) = P(z) - \delta(z-1)\int_0^1 dyP(y)$.

We see that just as in the dressed quark case, the singularity arising as $x \to 1$ from real gluon emission is canceled in the quark distribution function once the normalization condition is properly taken in to account. From this derivation we begin to recognize the lowest order term of the Altarelli-Parisi evolution equation.

VI. DISCUSSION

In this paper we have investigated the feasibility of analyzing deep inelastic structure functions in the Hamiltonian formulation. Our starting point has been the non-perturbative result obtained using equal-x^+ light-front current algebra and the BJL limit which clearly shows exact Bjorken scaling if we forget the non-trivial renormalization issues involved in the matrix element of the bilocal current sandwiched between hadronic states. In this preliminary study we have focused on the leading contribution (ignoring power suppressed terms) to the unpolarized structure function F_2. Since the *good* (+) component of the bilocal current is involved, expanding the hadronic state into multi-parton wavefunctions, we obtained the structure function $\frac{F_2}{x}$ as a sum of the squared multi-parton wavefunction integrated over longitudinal and transverse momenta. Taking into account the QCD interaction which connects the various wavefunctions and scaling violation comes in the picture. We took the light-front Hamiltonian and demonstrated how one could get logarithmic growth and the splitting functions in the context of a dressed quark. The probabilistic interpretation of the splitting function is very clear in this language. We also noted how $x \to 0$ singularity could be taken care of by the normalization of the state in this language. In the bound state calculations involving a meson, we clearly illustrated how factorization of the non-perturbative and perturbative parts in the structure function occurred in our approach. We have also illustrated how various coherent effects are power suppressed in the hard region which leads to the standard perturbative evolution of parton distributions.

It is of interest to carry out higher order calculations for the dressed quark to see explicitly how the running coupling constant comes into play in this formulation.

The ultimate test of the present formulation (success or failure) is in its application to the analysis of the so-called *higher-twist* structure functions. The longitudinal structure function F_L and the polarized structure function G_T currently attracts lot of attention. Even the questions of physical interpretation, factorization and evolution of these structure functions are under intense investigation and debate at the present time. We plan to study these problems in the near future in the formulation presented in this paper which keeps

close contact with physical intuitions of the pre-QCD parton picture while maintaining the complexities of QCD.

ACKNOWLEDGMENTS

We acknowledge discussions with the members of the Theory Group, Saha Institute of Nuclear Physics, and the members of the Nuclear Theory Group, Physics Department, Iowa State University. We also appreciate many useful conversations with Wei-Min Zhang. This work was supported in part by the U.S. Department of Energy under Grant No. DEFG02-87ER40371, Division of High Energy and Nuclear Physics.

APPENDIX A: NOTATIONS AND CONVENTIONS

We use the definitions

$$x^{\pm} = x^0 \pm x^3, \;\; x^{\perp} = (x^1, x^2). \tag{A1}$$

$$\gamma^{\pm} = \gamma^0 \pm \gamma^3. \tag{A2}$$

.

$$\psi^{\pm} = \Lambda^{\pm}\psi, \text{ with } \Lambda^{\pm} = \frac{1}{4}\gamma^{\mp}\gamma^{\pm}. \tag{A3}$$

The normalization of the state is

$$\langle P \mid P' \rangle = 2(2\pi)^3 P^+ \delta^3(P - P'). \tag{A4}$$

The volume elements

$$[dk] = \frac{dk^+ d^2k^{\perp}}{2(2\pi)^3\sqrt{k^+}}, \tag{A5}$$

$$\{dk\} = \frac{dk^+ d^2k^{\perp}}{\sqrt{2(2\pi)^3 k^+}}, \tag{A6}$$

and

$$(dk) = \frac{dk^+ d^2k^{\perp}}{2(2\pi)^3 k^+}. \tag{A7}$$

APPENDIX B: NORMALIZATION OF STATE

The normalization of the state $\mid P\rangle$ is

$$\langle P' \mid P\rangle = 2(2\pi)^3 P^+ \delta^3(P - P'). \tag{B1}$$

In the truncated space for the meson state, we have,

$$\begin{aligned}\sum_{\sigma_1,\sigma_2} \int dk_1^+ d^2k_1^\perp \int dk_2^+ d^2k_2^\perp \delta^3(P - k_1 - k_2) \mid \phi_2(P \mid k_1, \sigma_1; k_2, \sigma_2) \mid^2 \\ + \sum_{\sigma_1,\sigma_2,\lambda_3} \int dk_1^+ d^2k_1^\perp \int dk_2^+ d^2k_2^\perp dk_3^+ d^2k_3^\perp \delta^3(P - k_1 - k_2 - k_3) \\ \times \mid \phi_3(P \mid k_1, \sigma_1; k_2, \sigma_2, k_3, \lambda_3) \mid^2 = 1.\end{aligned} \tag{B2}$$

In terms of the amplitudes ψ_2, ψ_3, the normalization condition in the truncated Fock space sector reads

$$\begin{aligned}\sum_{\sigma_1,\sigma_2} \int dx_1 d^2\kappa_1^\perp \int dx_2 d^2\kappa_2^\perp \delta(1 - x_1 - x_2)\delta^2(\kappa_1^\perp + \kappa_2^\perp) \mid \psi_2^{\sigma_1,\sigma_2}(x_1, \kappa_1^\perp; x_2, \kappa_2^\perp) \mid^2 + \\ \sum_{\sigma_1,\sigma_2,\lambda_3} \int dx_1 d^2\kappa_1^\perp \int dx_2 d^2\kappa_2^\perp \int dx_3 d^2\kappa_3^\perp \delta(1 - x_1 - x_2 - x_3)\delta^2(\kappa_1^\perp + \kappa_2^\perp + \kappa_3^\perp) \\ \times \mid \psi_3^{\sigma_1,\sigma_2,\lambda_3}(x_1, \kappa_1^\perp; x_2, \kappa_2^\perp; x_3, \kappa_3^\perp) \mid^2 = 1.\end{aligned} \tag{B3}$$

APPENDIX C: OPERATOR DEFINITION OF GLUON DISTRIBUTION FUNCTION AND THE LONGITUDINAL MOMENTUM SUM RULE

Consider the conserved longitudinal momentum operator

$$P^+ = \frac{1}{2} \int dx^- d^2x^\perp \theta^{++} \tag{C1}$$

where

$$\theta^{++} = \theta_F^{++} + \theta_G^{++}. \tag{C2}$$

The fermion contribution

$$\theta_F^{++} = i\bar{\psi}\gamma^+\partial^+\psi \tag{C3}$$

and the gluon contribution

$$\theta_G^{++} = F^{+\nu a} F_\nu^{+a} \tag{C4}$$

where

$$F^{\mu\nu a} = \partial^\mu A^{\nu a} - \partial^\nu A^{\mu a} + g f^{abc} A^{\mu b} A^{\nu c}. \tag{C5}$$

From the definition

$$F_2^F(x) = \frac{x}{4\pi P^+} \int d\eta e^{-i\eta x} \langle P \mid [\bar{\psi}(y)\gamma^+\psi(0) - \bar{\psi}(0)\gamma^+\psi(y)] \mid P\rangle, \tag{C6}$$

with $\eta = \frac{1}{2}P^+y^-$, we have

$$\int dx F_2^F(x) = (\frac{1}{P^+})^2 \langle P \mid \theta_F^{++} \mid P\rangle. \tag{C7}$$

Formally, we can define the "gluon structure function" [34]

$$F_2^G(x) = \frac{1}{4\pi P^+} \int dy^- e^{-\frac{i}{2}P^+y^-x} \langle P \mid F^{+\nu a}(y^-) F_\nu^{+a}(0) \mid P\rangle. \tag{C8}$$

We have,

$$\int dx F_2^G(x) = (\frac{1}{P^+})^2 \langle P \mid \theta_G^{++} \mid P\rangle \tag{C9}$$

and the momentum sum rule

$$\int dx [F_2^F + F_2^G] = 1. \tag{C10}$$

Next we explicitly verify the longitudinal momentum sum rule in the truncated Fock space. The fermionic part of the longitudinal momentum operator

$$P_F^+ = \frac{1}{2} \int dx^- d^2x^\perp 2i(\psi^+)^\dagger \partial^+ \psi^+. \tag{C11}$$

Using the field expansion

$$\psi^+(x) = \sum_\lambda \chi_\lambda \int [dp][b(p,\lambda)e^{-ip.x} + d^\dagger(p,-\lambda)e^{ip.x}], \tag{C12}$$

$$P_q^+ = \sum_\lambda \int (dp) p^+ [b^\dagger(p,\lambda)b(p,\lambda) + d^\dagger(p,\lambda)d(p,\lambda)], \tag{C13}$$

with (dp) defined in appendix A.

In $A^{+a} = 0$ gauge, the gluonic part of the longitudinal momentum operator

$$P_G^+ = \frac{1}{2} \int dx^- d^2x^\perp \partial^+ A^{ja} \partial^+ A^{ja}. \tag{C14}$$

Using the field expansion

$$A^{ja} = \sum_\lambda \int (dq)[\epsilon_\lambda^j a^a(q,\lambda)e^{-iq.x} + (\epsilon_\lambda^j)^*(a^a)^\dagger(q,\lambda)e^{iq.x}], \tag{C15}$$

$$P_G^+ = \sum_\lambda \int (dq) q^+ (a^a)^\dagger(q,\lambda) a^a(q,\lambda). \tag{C16}$$

We have the relation

$$\langle P' \mid P^+ \mid P \rangle = [2(2\pi)^3 P^+ \delta^3(P-P')] P^+. \tag{C17}$$

Explicit evaluation in the truncated Fock space gives

$$\begin{aligned}
&\sum_{\sigma_1,\sigma_2} \int dx_1 d^2\kappa_1^\perp \int dx_2 d^2\kappa_2^\perp \delta(1-x_1-x_2)\delta^2(\kappa_1^\perp+\kappa_2^\perp) \\
&\qquad \times (x_1+x_2) \mid \psi_2^{\sigma_1,\sigma_2}(x_1,\kappa_1^\perp;x_2,\kappa_2^\perp) \mid^2 + \\
&\sum_{\sigma_1,\sigma_2,\lambda_3} \int dx_1 d^2\kappa_1^\perp \int dx_2 d^2\kappa_2^\perp \int dx_3 d^2\kappa_3^\perp \delta(1-x_1-x_2-x_3)\delta^2(\kappa_1^\perp+\kappa_2^\perp+\kappa_3^\perp) \\
&\qquad \times (x_1+x_2+x_3) \mid \psi_3^{\sigma_1,\sigma_2,\lambda_3}(x_1,\kappa_1^\perp;x_2,\kappa_2^\perp;x_3,\kappa_3^\perp) \mid^2 = 1,
\end{aligned} \tag{C18}$$

which is automatically satisfied because of the normalization condition given in Eq.(B3). Thus momentum fraction sum rule is trivially satisfied given the normalization condition on the amplitudes ψ_2, ψ_3 etc.

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